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Instability of circularly polarized electro-kinetic waves in magnetized ion-implanted semiconductor plasmas

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Abstract. Based on the hydrodynamical model of plasmas, an analytical investigation of a transverse electro-kinetic wave and the novel properties introduced due to the presence of negatively charged colloidal particles in a compensated, magnetized group III-V semiconductor plasma is presented. We derive a compact linear dispersion relation for left-hand circularly polarized electro-kinetic waves in an ion-implanted semiconductor plasma by using multi-fluid analysis and Maxwell's equations. This dispersion relation is used to study numerically wave phenomena and the resultant instability of the left-hand circularly polarized mode. It is found that the presence of charged colloids significantly modify the dispersion and absorption characteristics of possible modes even though colloidal particles on account of their heavy masses do not participate in wave propagation. One out of the two modes is always found to be spatially growing with the growth rate increasing with the electric field.

PACS. 52.35.Hr Electromagnetic waves (e.g., electron-cyclotron, Whistler, Bernstein, upper hybrid, lower hybrid) – 72.30.+q High-frequency effects; plasma effects – 61.72.Ww Doping and impurity implantation in other materials – 82.70.Dd Colloids

1 Introduction

At present, a large number of workers [1–3] are focusing their attention to investigate various interesting properties of implanted colloids formed within an ion-implanted semiconductor material. Recently, this medium has attracted renewed interest in finding the novel modes due to excitations of new branches of Alfven [4], electro-kinetics [5], electro-acoustic waves [6], instability of acousto-electric waves [7] and modifications in existing wave spectra in colloids-laden or ion-implanted semiconductor plasmas. The current trends in the field indicate that the presence of charged colloids have a strong influence on the characteristics of usual plasma wave modes, even at frequencies where colloidal grains do not participate in wave motion. In such cases, the colloids simply provide an immobile charge neutralizing background.

It is known that in the presence of a magnetic field, electromagnetic waves can propagate in media with high electrical conductivity even if the frequency of the wave is lower than the plasma frequency of the media. Hence for several years, the propagation of such waves has been the primary subject in the study of semiconductor plasma characteristics because one can evaluate important properties of the medium from the dispersion relation of these waves.

Although electromagnetic waves (circularly polarized transverse electromagnetic waves) have been extensively studied over the last three decades, there are still tremendous possibilities for further exploration and exploitation. Since random and static distributions of highly charged and massive colloids can change the dispersive properties of the medium, which may be why the propagating wave suffers strong modifications, the study of these waves in such a medium becomes important for a better understanding of the wave spectrum and the medium properties.

Motivated by the renewed interest in this growing field and the fascinating works of Ghosh et al. [4–7], we present in this paper the analytical investigations of dispersion and absorption characteristics of the fundamental energy carrying mode i.e. electro-kinetic waves in a magnetized semiconductor plasma medium laden with colloids, in which negatively charged colloids are assumed to be stationary forming a neutralizing background in the medium. As far as we know, no systematic attempt has yet been made towards studying this wave and its properties in the ion-implanted semiconducting medium.

The paper is organized in the following manner. The basic equations describing the phenomena are presented in Section 2. This section also deals with a complete

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theoretical formulation of the dispersion relation for the generated circularly polarized transverse electro-kinetic wave in the colloid-laden semiconductor plasmas using the multi-fluid plasma model. In Section 3, we present exhaustive numerical appreciations followed by a detailed discussion. Finally the important conclusions drawn from the study are given in Section 4.

2 Theoretical formulations

This section deals with the theoretical formulation of the dispersion relation. We have considered the hydrodynamical model of a homogeneous multi-component semiconductor plasma, consisting of drifting electrons, holes and non-drifting negatively charged colloids of infinite extent. The medium is considered to be a compensated III-V semiconductor sample immersed in a static magnetic field $\mathbf{B}_0$ pointing along the propagation direction (z-axis).

We have assumed that all the colloids are of uniform size and are smaller than both the wavelength under study and the carrier Debye radii; hence they can be treated as negatively charged point masses. The colloidal particles are assumed to be stationary by considering them too massive to respond to the considered perturbations. Then the condition for charge neutrality in plasma with negatively charged colloids is given by

$$n_{0h} = n_{0e} + z_d n_{0d}, \tag{1}$$

where n_α $(\alpha = e, h, d)$ are unperturbed number densities of electrons, holes and colloids, respectively. $z_d = q_d/e$ is the ratio of negative charges q_d residing on the colloidal grains to the charge e of the electrons.

It has been reported by Cramer and Vladimirov [8,9] that the presence of dust grains creates a charge imbalance in the complex plasma medium and modifies wave and instability phenomena. A similar phenomenon can also be expected to occur in a colloid-laden semiconductor plasma medium due to the sticking of electrons on the surface of the colloids, which in turn is responsible for the modification of as well as the excitation of various propagating waves.

In the present report we shall study the propagation of transverse electro-magnetic waves in the presence of drifting charges (electrons and holes) and stationary negatively charged colloids. The transverse wave $\exp\left[i\left(\omega t - kz\right)\right]$ is taken to be propagating in the z-direction [where ω and k are the frequency and wave number of the transverse electro-kinetic mode, respectively] in an infinite homogeneous medium having a relative dielectric constant ε_L. Electrons are drifting in the positive z-direction with velocity $\vec{\vartheta}_{0e}$ while holes are drifting in the negative z-direction with velocity $\vec{\vartheta}_{0h}$. This configuration $\mathbf{B}_0||k||\hat{z}$ favors the propagation of transverse waves that are circularly polarized in the xy-plane.

In the presence of transverse electromagnetic modes, the macroscopic state of the colloid-laden semiconductor plasma may be described by the linearized momentum equation for drifting electrons and holes, respectively, as

$$\frac{\partial \vec{\vartheta}_{1e}}{\partial t} + \vec{\vartheta}_{0e}\frac{\partial \vec{\vartheta}_{1e}}{\partial z} + \nu_e \vec{\vartheta}_{1e} = \frac{-e}{m_e}\left[\vec{E}_1 + \vec{\vartheta}_{0e}\times\vec{B}_1 + \vec{\vartheta}_{1e}\times\vec{B}_0\right], \tag{2}$$

$$\frac{\partial \vec{\vartheta}_{1h}}{\partial t} - \vec{\vartheta}_{0h}\frac{\partial \vec{\vartheta}_{1h}}{\partial z} + \nu_h \vec{\vartheta}_{1h} = \frac{e}{m_h}\left[\vec{E}_1 - \vec{\vartheta}_{0h}\times\vec{B}_1 + \vec{\vartheta}_{1h}\times\vec{B}_0\right]. \tag{3}$$

The propagation of the wave in the medium can be described by Maxwell's equation:

$$\nabla\times E = \frac{-\partial B}{\partial t}, \tag{4}$$

where m_α, ϑ_α and ν_α are the masses, velocities and momentum transfer collision frequencies of the species α $(= e, h)$. All the other symbols have their usual meanings. Here the subscripts 0 and 1 used in the above equations (2) and (3) represent the zero and first order quantities, respectively.

Following the procedure adopted by Steele and Vural [10], the components of the perturbed velocities for electrons and holes are

$$\vec{\vartheta}_{\pm e} = \frac{ie}{m_e}\frac{\left(\omega - k\vec{\vartheta}_{0e}\right)\vec{E}_\pm}{\omega\left[\omega - k\vec{\vartheta}_{0e} - i\nu_e \mp \omega_{ce}\right]}, \tag{5}$$

$$\vec{\vartheta}_{\pm h} = \frac{-ie}{m_h}\frac{\left(\omega + k\vec{\vartheta}_{0h}\right)\vec{E}_\pm}{\omega\left[\omega + k\vec{\vartheta}_{0h} - i\nu_h \pm \omega_{ch}\right]}, \tag{6}$$

where $\omega_{ce,h} = (eB_0/m_{e,h})$ is the electron/hole cyclotron frequency. In obtaining equations (5) and (6), we have taken $\vartheta_\pm = \vartheta_{1x} \pm i\vartheta_{1y}$, $B_\pm = B_{1x} \pm iB_{1y}$ and $E_\pm = E_{1x} \pm iE_{1y}$; the upper (+) and lower (−) signs correspond to the right-hand and left-hand circular polarizations, respectively. Both ϑ_{1x} and ϑ_{1y} represent the perturbed components of ϑ along x- and y-axes respectively.

The total perturbed part of the current densities in the compensated semiconductor plasma may be written as

$$\begin{aligned} J_\pm &= J_{\pm e} + J_{\pm h} \\ J_\pm &= \frac{-i\varepsilon_0\varepsilon_L\omega_{ph}^2 E_\pm}{\omega}\left[\frac{(\delta/\mu)\left(\omega - k\vartheta_{0e}\right)}{\left[\omega - k\vartheta_{0e} - i\nu_e \mp \omega_{ce}\right]} + \frac{\left(\omega + k\vartheta_{0h}\right)}{\left[\omega + k\vartheta_{0h} - i\nu_h \pm \omega_{ch}\right]}\right]. \end{aligned} \tag{7}$$

Here we have defined $\omega_{ph}^2 = e^2 n_{0h}/\varepsilon m_h$, the hole-plasma frequency, $\mu = m_e/m_h$, the electron to hole mass ratio and $\delta = n_{0e}/n_{0h}$, the charge imbalance parameter originating from the presence of negatively charged colloids in the medium. This parameter measures the charge imbalance in the plasma medium with the remainder of the negative charges residing on the colloidal particles, so that the total system remains charge neutral. It is also worth mentioning here that the charging of colloids causes a depletion of species of higher mobility (here electrons). Due to the neutrality condition it is possible to have $n_{0e} \ll n_{0h}$ in such plasmas, hence

$$\delta = \frac{n_{0e}}{n_{0h}} < 1. \quad (8)$$

Using the general wave equation given by

$$\hat{k} \times \hat{k} \times \vec{E}_{\pm} = i\omega\mu_0 \vec{J}_{\pm} - \frac{\omega^2}{c_L^2}\vec{E}_{\pm}, \quad (9)$$

and equation (7), the dispersion relation for transverse electromagnetic waves in the presence of a background of stationary negatively charged colloids, is obtained as

$$\varepsilon(\omega, k) = k^2 - \frac{\omega^2}{c^2}\left[\varepsilon_L - \frac{\omega_{ph}^2(\delta/\mu)(\omega - k\vartheta_{0e})\varepsilon_L}{\omega^2[\omega - k\vartheta_{0e} - i\nu_e \mp \omega_{ce}]} + \frac{\omega_{ph}^2(\omega + k\vartheta_{0h})\varepsilon_L}{\omega^2[\omega + k\vartheta_{0h} - i\nu_h \pm \omega_{ch}]}\right] = 0, \quad (10)$$

where $c_L = c/\sqrt{\varepsilon_L}$ is the velocity of light in the medium.

By employing equation (10) in a collision dominated or a low frequency regime $[\nu_{e,h} \gg (\omega \mp k\vartheta_{0e,h})]$, and by further assuming that ε_L is very much less than the other terms in the square bracket, leads to simplified dispersion equation as:

$$c^2k^2 = \left[-\frac{\omega_{ph}^2(\delta/\mu)(\omega - k\vartheta_{0e})\varepsilon_L}{[-i\nu_e \mp \omega_{ce}]} - \frac{\omega_{ph}^2(\omega + k\vartheta_{0h})\varepsilon_L}{[-i\nu_h \pm \omega_{ch}]}\right]. \quad (11)$$

By neglecting ε_L we have taken out the fast electromagnetic wave carried by dielectric medium but are still left with the electro-kinetic waves carried by electrons and holes in the dispersion equation, so that we can study the possible wave instability characteristics.

It may be inferred from equation (11) that for electrons alone, the left handed polarization gives a negative energy carrying wave and for holes alone it gives a positive energy carrying wave and vice versa for the right hand polarized wave.

Now we shall focus our attention on the principle point of this paper i.e. the instability characteristics of the circularly polarized transverse electro-kinetic waves in the colloid-laden semiconductor plasma medium.

For a left-handed polarization on considering only the lower sign of equation (11) the dispersion relation becomes

$$c^2k^2 = \left[-\frac{\omega_{ph}^2(\delta/\mu)(\omega - k\vartheta_{0e})\varepsilon_L}{[-i\nu_e + \omega_{ce}]} - \frac{\omega_{ph}^2(\omega + k\vartheta_{0h})\varepsilon_L}{[-i\nu_h - \omega_{ch}]}\right]. \quad (12)$$

Equation (12) may be written in the form of a polynomial in k as

$$k^2c^2\mu[-(\nu_e\nu_h + \omega_{ce}\omega_{ch}) + i(\nu_e\omega_{ch} - \nu_h\omega_{ce})] + k\omega_{ph}^2\varepsilon_L[(\vartheta_{0e}\omega_{ch}\delta + \omega_{ce}\vartheta_{0h}\mu) + i(\vartheta_{0e}\nu_h\delta - \vartheta_{0h}\nu_e\mu)] + \omega\omega_{ph}^2\varepsilon_L[(\omega_{ce}\mu - \omega_{ch}\delta) - i(\nu_h\delta + \nu_e\mu)] = 0. \quad (13)$$

It can be inferred from equation (13) that the left hand circularly polarized wave has two branches of propagation. This equation being second order can be solved numerically to study the dispersion and convective amplification characteristics by considering ω as real positive quantity and k complex.

It is a well-established fact that a positive energy-carrying wave can interact only with a negative energy-carrying wave of the same polarization and vice versa. Also it has been well documented in the literature [10] that the right-hand polarized wave does not have any positive value of ω for $k > 0$ and so it is not involved in any fruitful interaction. Hence in the present paper we only concentrate on interactions involving a left-hand circularly polarized wave with the medium.

3 Results and discussions

The dispersion relations derived in the preceding section can be employed to study dispersion and convective absorption characteristics of circularly polarized electro-kinetic waves in an ion-implanted group III-V semiconductor plasma.

We have considered that the first order perturbation is of the form $\exp[i(\omega t - kz)]$ and so the wave may be spatially growing in the direction of propagation when the imaginary part of the wave vector $k_i > 0$, and the wave is extracting power from the medium which causes convective instability. On the other hand the wave may be spatially decaying in the direction of propagation when $k_i < 0$ then the medium absorbs power from the wave so that the wave suffers attenuation.

To have a numerical appreciation of the results obtained in the previous section, the following set of parameters for the compensated InSb semiconductor medium has been used: $m_e = 0.014m_0$, m_0 being the free electron mass, $m_h = 0.4m_e$, $\varepsilon_L = 17.5$, $n_{0e} \approx n_{0h} = 10^{21}$ m^{-3}, $\nu_e = 3.5 \times 10^{11}$ s^{-1}, $\nu_h = 4.4 \times 10^{11}$ s^{-1}.

For the left hand circularly polarized electro-kinetic wave, the results are displayed in the form of graphs in Figures 1–8.

Figures 1–4 illustrate the dispersion and absorption characteristics of the two modes with δ as the variable parameter at constant $E_0 = 10^3$ Vm^{-1} and $B_0 = 0.1$ T. In the absence of any implanted ions, the plasma has equal number densities of electrons and holes ($\delta = 1$). Figures 1 and 2 display the variation of the phase constant (k_r) and gain coefficients (k_i) of the first mode of the transverse electro-kinetic wave with wave frequency ω. Figure 1 suggests that the qualitative nature of the dispersion is identical for the chosen values of δ. It may be inferred from

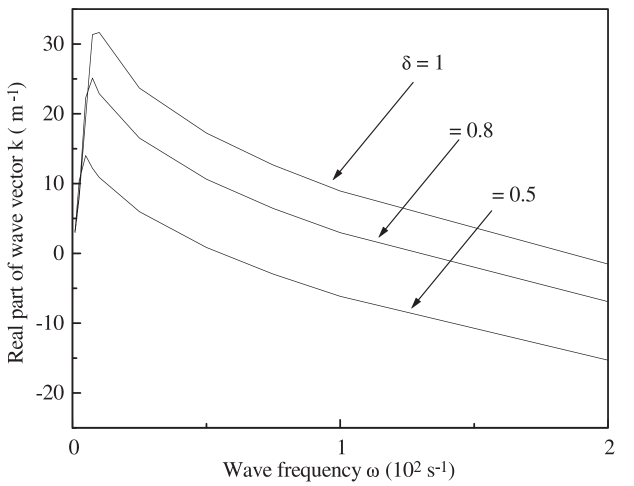

Fig. 1. Variation of the real part of the wave vector k of the first mode with the wave frequency ω, using δ as the variable parameter, at $E_0 = 10^3$ Vm^{-1} and $B_0 = 0.1$ T.

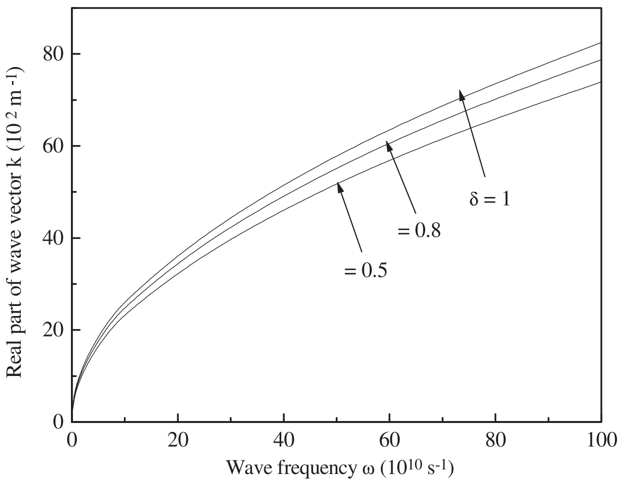

Fig. 3. Variation of the real part of the wave vector k of the second mode with the wave frequency ω, using δ as the variable parameter, at $E_0 = 10^3$ Vm^{-1} and $B_0 = 0.1$ T.

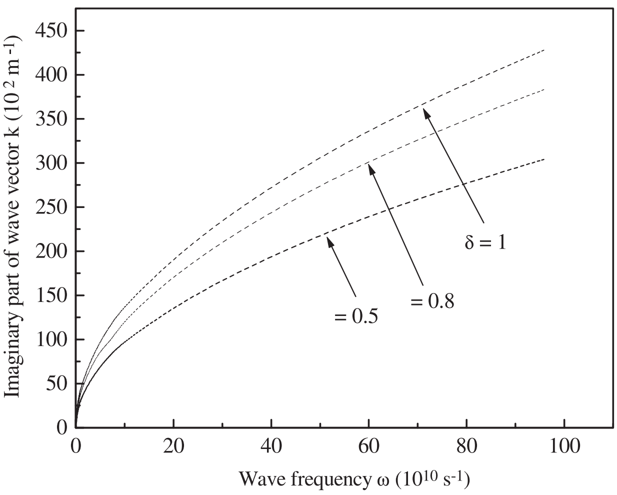

Fig. 2. Variation of the imaginary part of the wave vector k of the first mode with the wave frequency ω, using δ as the variable parameter, at $E_0 = 10^3$ Vm^{-1} and $B_0 = 0.1$ T.

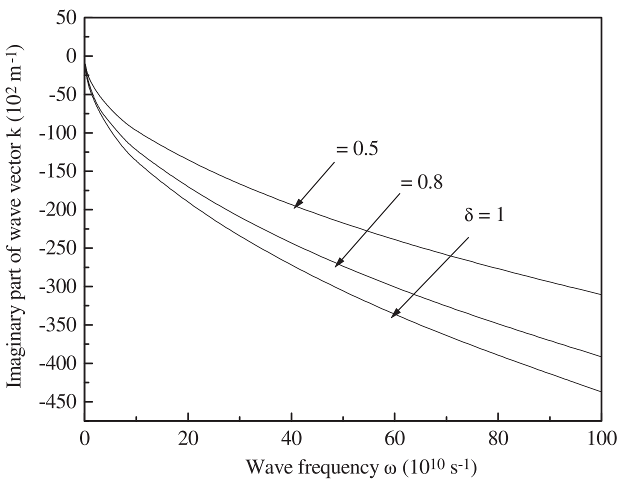

Fig. 4. Variation of the imaginary part of the wave vector k of the second mode with the wave frequency ω, using δ as the variable parameter, at $E_0 = 10^3$ Vm^{-1} and $B_0 = 0.1$ T.

Figure 1 that at very low frequency ($\omega \leq 10^2$ s^{-1}) this mode is propagating along the $+z$-direction and beyond the value of δ it changes its direction of propagation. The magnitude of the phase constant first increases with the increase in ω before achieving a maximum at a critical value of ω (say ω_{cr}) and then starts decreasing as ω continues to increase. This critical value of ω increases with an increment in δ (for $\delta = 0.5$, $\omega_{cr} = 0.0542 \times 10^2$ s^{-1}; for $\delta = 1$, $\omega_{cr} = 0.095 \times 10^2$ s^{-1}). As one goes on increasing ω, the phase constant becomes zero and beyond that one obtains propagation in opposite direction. Here the frequency bandwidth, which allows propagation of the wave, increases as the charge imbalance parameter increases.

Figure 2 infers that this mode is always growing in nature ($k_i > 0$) and that the growth rate increases with increasing ω. Here, it can also be seen that the growth rate is maximum for $\delta = 1$ and decreases as the negative charge concentration on the colloids increases (the magnitude of δ decreases).

Figures 3 and 4 depict the respective dependence of the real and imaginary parts of the wave vector k of the second mode of the transverse electro-kinetic wave on the wave frequency ω. This mode is found to be always decaying and propagating towards the $+z$-direction. As the charges on the colloids increases (i.e. δ decreases), the phase velocity decreases while the absorption of the mode increases. It can also be noticed from figures that in the higher frequency regime the wave starts propagating with a relatively greater phase speed and at the same time also shows strong attenuation characteristics.

Hence from the above discussion it is clear that in the presence of negatively charged stationary colloids, the two modes are found to be counter-propagating in nature with

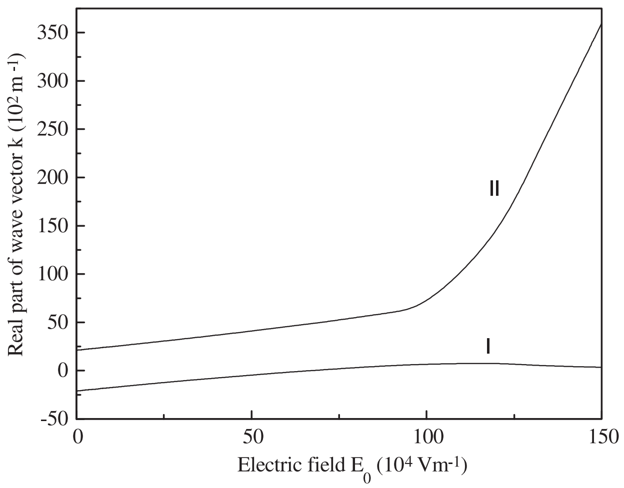

Fig. 5. Variation of the real part of the wave vector k of the two modes with E_0 at $\delta = 0.5$, $B_0 = 0.1$ T and $\omega = 10^{11}$ s^{-1}.

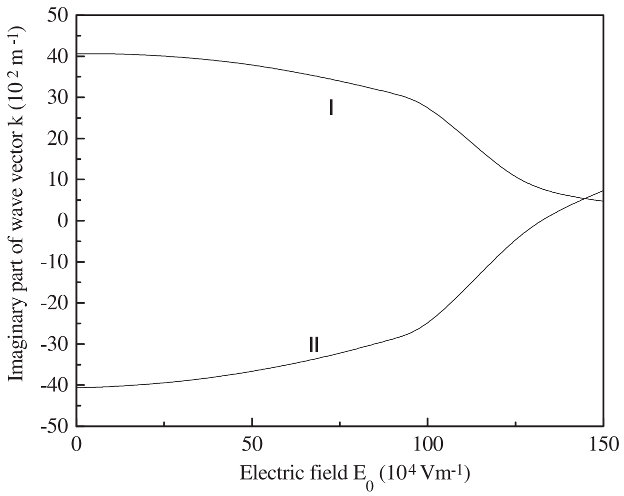

Fig. 6. Variation of the imaginary part of the wave vector k of the two modes with E_0 at $\delta = 0.5$, $B_0 = 0.1$ T and $\omega = 10^{11}$ s^{-1}.

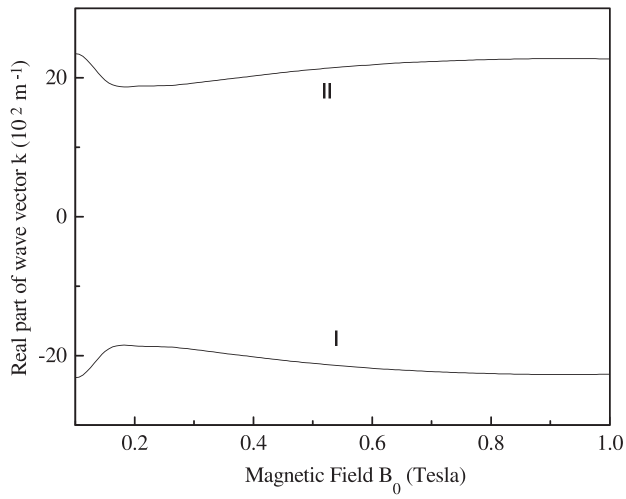

Fig. 7. Variation of the real part of the wave vector k of the two modes with B_0 at $\delta = 0.5$, $E_0 = 10^3$ Vm^{-1} and $\omega = 10^{11}$ s^{-1}.

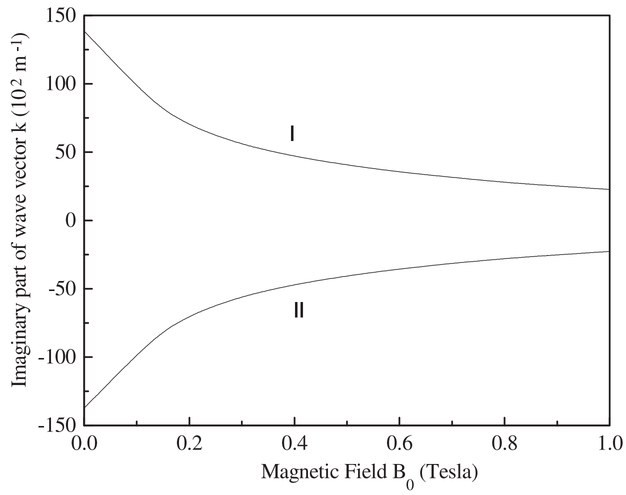

Fig. 8. Variation of the imaginary part of the wave vector k of the two modes with B_0 at $\delta = 0.5$, $E_0 = 10^3$ Vm^{-1} and $\omega = 10^{11}$ s^{-1}.

increasing absorption/attenuation constants towards the higher frequency range.

The variations of k_r and k_i with the strength of the electric field E_0 for the first and second modes are displayed in Figures 5 and 6, respectively, at $B_0 = 0.5$ T, $\delta = 0.5$ and $\omega = 10^{11}$ s^{-1}. Figure 5 infers that initially at $E_0 < 6.682 \times 10^5$ Vm^{-1}, the first mode is propagating towards the negative z-direction. Further on as the strength of electric field increases, the value of the phase constant increases and touches zero at $E_0 \approx 6.682 \times 10^5$ Vm^{-1}. Beyond this value, the phase constant slowly increases causing the mode to propagate in the opposite direction $(+z)$ and then saturates for greater E_0. On the other hand, the second mode is always propagating towards the positive direction for all values of E_0. Initially, the phase speed of this mode slowly increases but for $E_0 > 9.3 \times 10^5$ Vm^{-1} it starts increasing rapidly. Figure 6 illustrates that both these modes have an opposing variation in the presence of an applied electric field i.e. the first mode shows an amplification characteristic whose growth rate decreases with increasing $\mathbf{E}_0$ while the attenuation constant of the second mode increases with E_0. At $E_0 \approx 1.32 \times 10^6$ Vm^{-1} it becomes zero before again increasing. At $E_0 \approx 1.4 \times 10^6$ Vm^{-1} the two modes cross over. Beyond $E_0 \approx 1.4 \times 10^6$ Vm^{-1} the second mode maintains its increasing trend and the first one shows a minimum value of k_i(growth rate) for higher E_0.

In Figures 7 and 8, we have plotted k_r and k_i for the two transverse electro-kinetic modes against the variations of the magnetic field B_0 at constant $E_0 = 10^3$ Vm^{-1}, $\delta = 0.5$ and $\omega = 10^{11}$ s^{-1}. It is clear from Figure 7 that both the modes are purely counter-propagating modes in the presence of B_0. As the strength of the magnetic field B_0 increases in the medium, the phase velocities of the former/later mode initially ($B_0 < 0.2$ T) increases/decreases. On further increasing the value of B_0, the modes now start slowly decreasing/increasing before nearly saturating out.

From Figure 8, one infers that the first mode is always growing in nature while the second one shows attenuation characteristics. The growth rates of both these modes vary parabolically with the increasing strength of magnetic field i.e. the growth rates of first/second mode decreases/increases with increasing B_0.

4 Conclusions

In summary, we have analytically investigated the possibility of excitation and convective amplification of left-hand circularly polarized electro-kinetic waves in an ion-implanted group III-V semiconductor plasma. We have shown that the dispersion and absorption properties of these waves are strongly modified when some of the colloids acquire negative charges and their role becomes increasingly effective as the number of charges stuck on them increases. Moreover, it is also shown that there exists one mode (first) out of the two, which leads to instability or spatial growth in the direction of propagation and other mode leads to spatial decay or attenuation. The growth rate of spatially growing mode increases rapidly at higher values of the electric field.

Thus it is hoped that the present fundamental study will lead to a better understanding of the interaction of a left-hand circularly polarized transverse electro-kinetic wave with a magnetized dense III-V semiconductor and can be put to use in various interesting applications.

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